PERGAMON

International Journal of Solids and Structures 37 (2000) 6173–6174

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Letter to the editor

Comments on "Exact solutions for the plane problem in piezoelectric materials with an elliptic or a crack", *International Journal of Solids and Structures*, Vol. 36, No. 17, pp. 2527–2540 (1999) by Cun-Fa Gao and Wei-Xun Fan

The problem of an infinite piezoelectric plane with a center void/crack has received considerable attention in literature. Gao and Fan (1999) revisited this topic and presented an interesting discussion. Their analytical solutions based on Sosa and Khutoryansky (1996) are rather concise and explicit. The discusser would like to make the following comments.

1. Eqs. (38)–(40) involving solutions to a hole are incorrect. Using the conditions given by Eq. (A6), it can be proved that c_R defined in Eq. (37a) is zero. As a result, D_1^0 and D_2^0 can be solved from Eqs. (36) and (18).
2. By collapsing an elliptical hole to a crack, Gao and Fan (1999) drew their main conclusions: the electroelastic field in piezoelectrics is irrelevant to the dielectric constant of the air inside a crack; uniform electrical loading has no effect on the field singularities. Such conclusions are based on the assumption that the length of semi-axis b of the hole is zero. Starting from this assumption, the above conclusions can be reached alternatively by physical consideration. For a piezoelectric medium containing an elliptical hole, the electrical conditions on the air–piezoelectrical interface are the continuities of the tangential component of the electric field (or electrical potential) and the normal component of the electric displacement (Parton and Kudryavtsev, 1988). When a hole is reduced to a crack ($b = 0$), the air phase is physically absent. Consequently, the permeable crack boundary condition proposed by Polovinkina and Ulitko (1978) and Mikhailov and Parton (1990), i.e. the electrical potential and the normal component of the electric displacement are continuous across the crack faces, is automatically established. Based on the condition of a permeable crack boundary, Gao and Fan's conclusions (1999) are obvious without a detail analysis. Although the electrical boundary condition on crack faces is a key issue in the fracture mechanics of piezoelectrics, no consensus has been reached on this issue. Suo et al. (1992) argued that the permeable crack boundary is not realistic, as there will be a potential drop across crack faces. Hao and Shen (1994) proposed a new electrical boundary condition in which the electrical permeability of the air in a crack is considered. The real condition on crack faces still awaits experimental verification.
3. The conditions given by Eq. (A6) are not valid for all transversely isotropic piezoelectric materials. An example of this is the PZT-6B ceramics with material constants given by Rajapakse (1997). For PZT-6B, $\mu_1 = 2.1014i$, $\mu_2 = 1.0143i$ and $\mu_3 = 0.5176i$. In fact, Eq. (A6) is not necessary in the study. Simplifying Eq. (44) to Eq. (45) makes no difference to the conclusions of the paper. Also D_1^0 and D_2^0 can be solved from Eqs. (36) and (18) without using the condition of vanishing c_R.

0020-7683/00/$ - see front matter
PII: S0020-7683(99)00261-9

References

Gao, C.-F., Fan, W.-X., 1999. Exact solutions for the plane problem in piezoelectric materials with an elliptic or a crack. International Journal of Solids and Structures 36, 2527–2540.

Hao, T.-H., Shen, Z.-Y., 1994. A new electric boundary condition of electric fracture mechanics and its applications. Engineering Fracture Mechanics 47, 793–802.

Mikhailov, G.K., Parton, V.Z., 1990. Electromagnetoelasticity. Hemisphere, New York.

Parton, V.Z., Kudryavtsev, B.A., 1988. Electromagnetoelasticity. Gordon & Breach, New York.

Polovinkina, I.B., Ulitko, A.F., 1978. On the equilibrium of piezoceramic bodies containing cracks. TN 18, 10–17.

Rajapakse, R.K.N.D., 1997. Plane stress/strain solutions for piezoelectric solids. Composites 28B, 385–396.

Sosa, H., Khutoryansky, N., 1996. New developments concerning piezoelectric materials with defects. International Journal of Solids and Structures 33, 3399–3414.

Suo, Z., Kuo, C.M., Barnett, D.M., Willis, J.R., 1992. Fracture mechanics for piezoelectric ceramics. Journal of the Mechanics and Physics of Solids 40, 739–765.

X.-L. Xu

Department of Civil and Geological Engineering, University of Manitoba, Winnipeg, R3T 5V6, Canada

E-mail address: umxux@cc.umanitoba.ca

1 July 1999

Fax: +1-204-269-4863.